Our Fireball Radius Height of Burst Calculation is 75 Years Old:
is it Time for an Update?

By: Col. Jeff Kendellen

Introduction

For every single nuclear weapon considered for employment—starting with Japan in 1945 to today at US Strategic Command—planners will address the question: will this detonation produce fallout? The ‘yes’ or ‘no’ answer to this question has traditionally relied upon an equation with known significant limitations. I propose a paradigm shift. It is time to update the way our community calculates heights of burst when considering fallout. In this paper, I will present important aspects of the nuclear fireball, a historical review of fireball radii calculations, and introduce some fireball and ground interaction errors and misconceptions. And finally, I challenge the Funcational Area (FA)52 community to start a ‘scientific revolution’ to critically evaluate and update our current paradigm to serve our community better.[1]

Before we dissect the fireball, we must reflect on important and meaningful variations on the question asked above: will this detonation produce fallout? Another similar question is: will the fireball touch the ground? Perhaps more important to Army FA52 Officers and military planners is the question: will this detonation produce *militarily significant* fallout?

The Fireball

With the foci and questions above in mind, I will describe the very basic elements of the fireball creation. When a nuclear weapon detonates, the fissile material is consumed in small fractions of a microsecond creating extreme temperatures (millions of degrees Celsius) and pressures (hundreds of gigapascals). These extremes generated by the bomb energy are caused primarily by x-rays radiating outwards, interacting with and depositing energy in the bomb casing materials and with the nitrogen, oxygen, and other minor components of the atmosphere. The elements and molecules in the vicinity essentially fully disassociate and ionize. These close-by x-ray interactions contribute to fireball growth through the generation of a quasi-spherical volume of incredibly hot and dense plasma. This is the “initial fireball.”[2] “The fireball subsequently expands through a combination of radiative and hydrodynamic processes.”[3] The expansion of the initial volume of plasma occurs because of the drastic increase in mean free path of follow-on x-rays.[4] These x-rays travel farther extending the sphere outwards because of more atmospheric interactions and energy deposition thereby creating the later growing hot and dense ionized plasma fireball.

The expansion of the fireball generates a “intense shock wave (thin region of highly compressed air)... formed at the surface, which expands outwards at a high velocity.”[5] Eventually, the x-rays stop radiating and the hot dense plasma, the excited molecules, and the environment around the fireball all must find balance and equilibrium. Part of this process as the fireball stops expanding creates a transition from a radiation wave to a shock wave—referred to as “hydrodynamic separation.”[6] This is the breakaway point and aligns approximately with the fireball’s maximum size.[7]

Historical Review of Radius Calculations

With the basic elements of the fireball discussed, the next section will review various sources, equations, and calculations for fireball radius. Sir Geoffrey Taylor's initial fireball radius work started in 1941 when he was tasked by the British Civil Defence Research Committee to mathematically evaluate a "bomb in which a very large amount of energy would be released by nuclear fission."[8] He completed his task in June 1941, but his work was classified until 1950.[9] In 1950, he published his report with the addition of an application of his mathematical work applied to the 1945 Trinity test.[10] Sir Taylor took film and still photographs released by the US Atomic Energy Commission (and printed by Life Magazine) of the growing fireball radius over time and was able to fine tune his assumptions and novel constants within his previous work to calculate a yield of 16.8 kilotons.[11] His yield estimate is surprisingly close to the actual yield of 21 kilotons.[12] Taylor's Equation is given as equation (1) where W is yield in kilotons, and ρ0 is atmospheric pressure in kg / m3.

$$Radius = \left(\frac{t}{0.926}\right)^{2/5} \frac{W^{1/5}}{\rho_0{}^{1/5}} \qquad (1)$$

The next equation is perhaps the most well-known. Glasstone's (and later Dolan's) equation provided another classic equation within *The Effects of Nuclear Weapons* to determine where the height of burst "ceases to be a serious problem."[13] This is given by equation (2) where W is yield in kilotons.

$$\begin{aligned}\text{Height of Burst (ft)} &= 180 * W^{0.4}\\ \text{Height of Burst (m)} &= 54.9 * W^{0.4} \qquad (2)\end{aligned}$$

It is important to stress, given the focus of this paper, that Glasstone and Dolan provide a substantial +/-30% error and specifically state heights of burst calculated using equation (2) may still produce fallout, but the fallout will "be small enough to be tolerable under emergency conditions."[14]

With respect to Glasstone's (and Dolan's) work, there are important editorial observations that should be pointed out. Equation (2) within the 1977 'third edition' exists in the same form within earlier versions of *The Effects of Nuclear Weapons* which were released in February 1964 and April 1962 (which were solely attributed to Glasstone as editor). Interestingly, Glasstone's 1957 version has the same equation (2), but the '+/-30%' and follow-on discussion regarding 'emergency conditions' was not mentioned. Instead, Glasstone provides a separate equation, only published in the 1957 version, for the "maximum size of the luminous fireball" which is given by equation (3) where W is yield in kilotons.[15]

$$\begin{aligned}\text{Radius(ft)} &= 230 * W^{0.4}\\ \text{Radius(m)} &= 70 * W^{0.4} \qquad (3)\end{aligned}$$

The reader should immediately note any output from equation (3) will be greater than equation (2) which perhaps speaks to Glasstone's 1957 view of 'negligible fallout.' Glasstone's (and the broader community's) data at the time was limited to sixty-one non-contact above ground nuclear weapon tests starting with Trinity in 1945 and ending with Operation Redwing in 1956.[16] Because equation (3) did not survive for future printings of *The Effects of Nuclear Weapons*, and future prints did include the '+/- 30%,' it is likely Glasstone reevaluated their stance to view fallout more conservatively when considering heights of burst and resultant fallout.

An interesting mathematical point about equation (3) is it is 27.8% more than calculations in equation (2). It is likely that while equation (3) was eliminated after the 1957 version, it was effectively retained in the 1962, 1964, and 1977 versions by rounding up and utilized as the '+30%' within the '+/- 30%.' A graph view of this point is clearly visible in Figure 1.

Another equation for fallout safe heights of burst is presented within the Staff Officers Field Manual *Nuclear Weapon Employment* FM 101-31-1 published in February 1963. Within this text, the authors provide two equations based on yield.[17] They can be seen below. Again, W is yield in kilotons.

$$HOB_{fs}(m) = 30 * W^{1/3} \text{ when the yield is} \leq 100 \text{ kilotons.} \qquad (4)$$

$$HOB_{fs}(m) = 56 * W^{1/3} \text{ when yield is} > 100 \text{ kilotons.} \qquad (5)$$

Equation (5) tracks reasonably well with the -30% version of equation (2) as seen in Figure 1. Between 105 kilotons and 500 kilotons the two never diverge more than 6%. Equations (4) and (5) are added to the buffer distance which is shown in equation (6) and accounts for weapon delivery inaccuracy. The inaccuracy in this case is not concerned with circular error probability, but instead the probable error in height (PE_h).

$$\text{Buffer distance} = 3.5 * \text{Probable Error Height} = 3.5 * PE_h \qquad (6)$$

$$HOB = HOB_{fs} + 3.5\ PE_h \qquad (7)$$

PE_h is a weapon system dependent error that also differs for how weapons are employed. Meaning, a certain nuclear capable artillery may have a different

PE_h for targets at different ranges which would be given in a table. The buffer distance creates assurances that 99% of the time, the weapon will detonate above the height calculated in equation (7).[18] Later versions of this Field Manual stopped using equations (4) and (5) and instead used heights of burst and PE_h from classified tables. Operators were expected to add the two together when planning weapon employment.

The next equations are primarily different from the previous equations in that they consider atmospheric pressure. The first is equation (8) contained within Ernest Bauer's 1990 report "Physics of High-Temperature Air – Part II Applications."[19] Radius is in kilometers, W is yield in megatons, ρ is air density at burst altitude, and po is air density at sea level.

$$Radius(t) = 0.85\,W^{0.33}\left(\frac{\rho_o}{\rho}\right)^{0.3} + 0.037W^{0.3}\left(\frac{\rho_o}{\rho}\right)^{0.2}(t - t^*) \quad (8)$$

Equation (9) comes from Dr. Hans Bethe's report "Theory of the Fireball" published in 1964.[20]

$$Radius = 0.78\,W^{\frac{1}{3}}\,(T')^{-0.1}\,\rho^{\frac{-1}{4}} \quad (9)$$

W is yield in megatons, ρ is pressure in bars, 'T′ is equal to 0.92, and radius is in kilometers. Dr. Bethe's equation, as shown in Figure 1, is a fair approximation to the output of equation (2).

Due to Distribution Statement restrictions, fireball equations from John Northrop's *Handbook of Nuclear Weapon Effects* and Charles J. Bridgman's *Introduction to the Physics of Nuclear Weapon Effects*, cannot be recorded here. It can be said that Northrop's equation plotted is less than the plot created using equation (2). And Bridgman's equation plotted out is less than the -30% Glasstone and Dolan plot. Both curves are of similar shape.

Aside from equations in books, manuals, and reports, there are number of tools that calculate fallout safe heights of burst. One notable software program in use currently is called *Hazard Prediction and Assessment Capability* which models nuclear weapon detonations by creating a source term and using real-world weather to model the lofting of material and subsequent fallout deposition in the local area. This software simply uses Glasstone and Dolan's equation (2) with no error applied.[21] US Strategic Command planners also use tools and software based on Glasstone and Dolan's classic equation (2). Planners use equation (2) as the "analytic starting point" while also incorporating weapon delivery system errors.[22]

The last tool I will mention, is somewhat of a collector's item—the *Nuclear Bombs Effects Computer*—often call a whiz-wheel which was included at the back of some copies of Glasstone and Dolan. Naturally, the calculations for fireball and ground interaction are based on Glasstone and Dolan's equation (2).

Misconceptions and Errors

These sources and equations should offer a perspective on how important the fireball/fallout

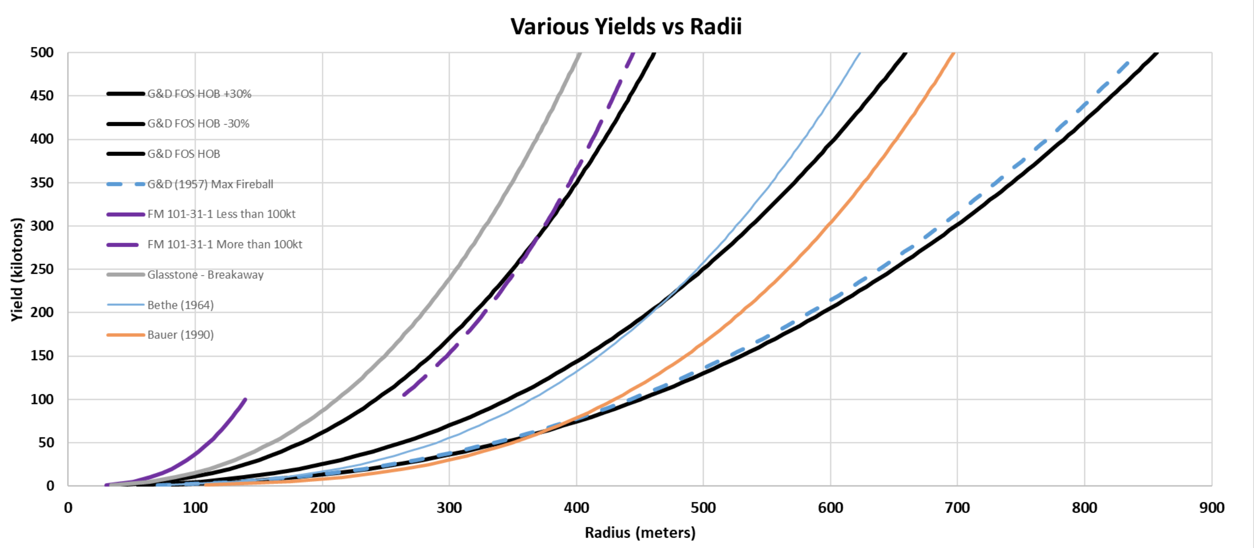


FIGURE 1. is a representation of the equations listed in the section above.

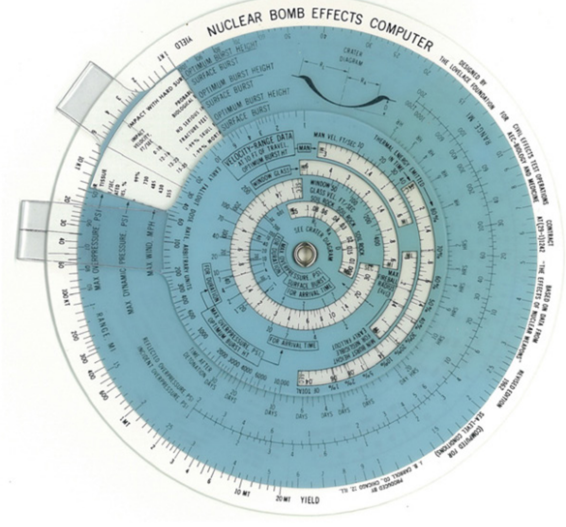


FIGURE 2. The Nuclear Bomb Effects Computer.

question was and is. Yet the lack of fidelity – both in the number of ways to calculate fireball radius, the errors given for these ways, and the changes over time—should cause concern. In the next section, the paper will attempt to add to that concern by providing some obvious, and perhaps not-so-obvious challenges, misconceptions, and errors when answering the fireball/fallout question(s).

Nuclear Weapons are Not Perfect

No nuclear weapon detonation is ever perfect. Films of fireballs prove this with what Harold Brode describes as fireballs that "lack symmetry or sphericity. There are frequently relatively large blisters and bright spots associated with the glassy looking shock front. These are evidence of the non-ideal nature of fireballs or rather features of nuclear explosions, which are thrown at high velocities against the back of the shock wave."[23] Brode elaborates further by stating radius calculations should be based "on x-ray yield fraction rather than total yield, since this fraction can change both as a function of the yield and as a function of the bomb design."[24] Glasstone goes even further on this idea by writing: "numerical values given in this book are not—and cannot be—exact. They must inevitably include a substantial margin of error. Furthermore, two weapons of different design may have the same explosive energy yield, but the effects could be markedly different."[25] Beyond the fireball itself, the associated delivery systems have errors as articulated above within the discussion on buffer distances and probable error in heights.

Reentry Body/Vehicle Body Velocity

Another consideration is the distinction between testing and real-world employment. In practice, heights of burst calculations were developed using films from above ground tests where the nuclear detonation was nearly stationary (e.g., using towers or balloons). Obviously, in these test detonations, collecting data was important which necessitated a quasi-static test subject. These stationary detonations are the antithesis of the extreme velocities experienced by reentry bodies or reentry vehicles. The reentry bodies/vehicles attain maximum velocities ranging from 4,500 to 6,000 m/s which equates to Mach 13 to 17.[26] By assuming Mach 17 as a worst case, what do back of the envelope calculations tell us about this velocity on fireball creation?

First, it is important to note the energy released from the fissile fuel occurs very quickly, on the order of ~100 shakes or ~1 microsecond.[27] Even when traveling Mach 17, the weapon only moves about ~1 centimeter while the weapon fuel is expended. X-rays and gamma rays are traveling at the speed of light (2.99x108 m/s) outward, but the fireball itself grows over seconds to about 10 seconds for megaton explosions.[28] An interesting consideration is: does the fireball have inertia and does the inertia contribute to fireball deformations and/or fireball/fallout errors?

The author had hopes that nuclear weapon testing would elucidate what happens during a detonation at ~Mach 17, but the single above ground test of a ballistic missile was not associated with an "experimental program."[29] This one test of a ballistic missile with a nuclear warhead was during Operation Dominic under test Frigate Bird. During Frigate Bird, a 200-1000 kt weapon was launched the USS Ethan Allen on May 6, 1962.[30] Unclassified videos of the detonation do not contain sufficient detail to discern any visual conclusions about the fireball or deformation.

To answer the question above, the fireball will certainly not travel ~6,000 meters over one second along the flight path of the reentry body/vehicle. Similarly, the velocity/path will not appreciably deform the fireball or the fireball development. The reasoning is as follows: The x-rays are traveling at the speed of light which dwarfs the speed of the reentry vehicle/body. In other words, the bomb will detonate and the x-rays will be deposited generating the growing fireball with little to no impact from the Mach 17 velocity.

Does a Mushroom Cloud = Radioactive Fallout?

Within this topic, I will address both a misconception and a factor that further burdens fireball/fallout error. Here, the question is: does the creation of a mushroom cloud mean the fireball touched the ground? As presented, this is a trick question that happens to be well covered by in a 2020 article entitled “Fallout Cloud Regimes.” The question above is a trick question because there are cases where the fireball does not touch the ground (verified visually during above ground tests), but “dust [can be] drawn into the fireball during the initial fireball phase, quenching part of the plasma in the fireball.”[31] The authors list multiple tests where the fireball does not touch the ground, yet created local and downwind radioactive fallout. These tests are summarized below with comparisons to Glasstone and Dolan’s equation (2) along with +/-30%.

What Figure 3 shows is the Buster Jangle Easy test which was done at a height of burst 85% more than Glasstone and Dolan’s equation (2) value of 217 meters, and yet still produced fallout. Grable, where the height of burst was nearly equal to the equation (2) calculation, also produced fallout. The authors also stress in their paper: “fallout-free height of burst, as frequently included in discussion of nuclear effects literature … [is] something of a misnomer.”[32]

Fireball Deformation

In this section, I will elaborate on a special aspect of the previous errors concerning mushroom clouds. It is the case that the fireball can also be deformed by the reflected ground shock. FM 101-31-1 states: “If the weapon is burst in the air close to the ground, the blast wave will reach the ground, be reflected, and flatten the bottom of the fireball before it has reached maximum size.[33]

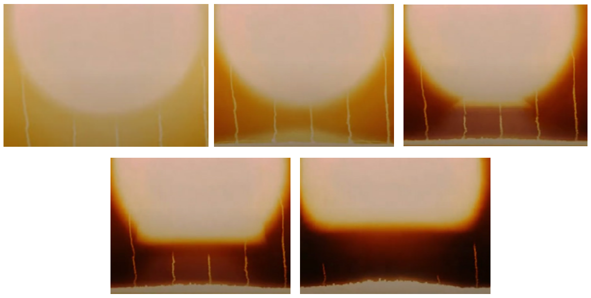


FIGURE 4. Still images from an unidentified nuclear weapon test near the ground. Extracted from the Nuclear Science and Engineering Research Center video: *A Review of Prompt Effects*.

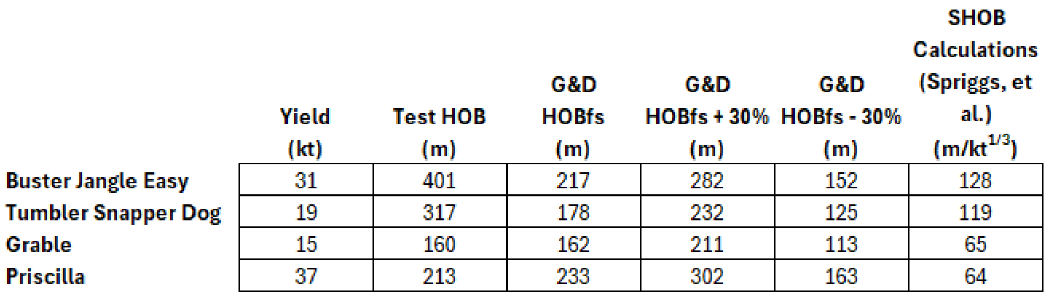
This effect, when seen visually in Figure 4, is assistive in helping one understand fireball growth, development and breakaway. At this point, the fireball is no longer growing appreciably, but can still be influenced by strong forces, such as from *its own* reflected shockwave.

Conclusion and a Challenge

Given the history, the stated errors, and potential misconceptions—Is it time for to update Glasstone and Dolan’s classic equation?

Yes!

To make this point simply, imagine the new car you just bought had a manufacturer stated highway range of 400 miles. Only for it to run out of gas at 280 miles. Where else in life or work do we just accept a +/- 30% error? Furthermore, an accurate starting point *is the source term* feeding follow-on tremendously important analysis. Without an accurate source term, our processes are ‘garbage in, garbage out.’



	Yield (kt)	Test HOB (m)	G&D HOBfs (m)	G&D HOBfs + 30% (m)	G&D HOBfs - 30% (m)	SHOB Calculations (Spriggs, et al.) (m/kt$^{1/3}$)
Buster Jangle Easy	31	401	217	282	152	128
Tumbler Snapper Dog	19	317	178	232	125	119
Grable	15	160	162	211	113	65
Priscilla	37	213	233	302	163	64

FIGURE 3. The summaries of relevant test data, equation (2) calculations, and Scaled Height of Burst calculations from Spriggs, et al.

I challenge the FA52 community to revolutionize how we—as a community charged with keeping our fellow Soldiers safe from contaminated fallout—calculate fallout safe heights of burst at US Strategic Command and the US Army Nuclear and Countering Weapons of Mass Destruction Agency. This will start with gathering the best data offered within Effects Manual-1, the research community, and recent scholarly work on nuclear weapon testing film digitization. Whatever the equation ends up being, and it might not even be much different from Glasstone and Dolan's equation (2), at least we will know the new answers have a solid foundation based on extensive data. Or we—as a community—can continue obdurately employing an equation from 1977, that was carried over from 1967 and 1962; copied from the 1957 version; and adopted from a 1950 manuscript 75 years ago after only five air burst nuclear weapons tests.[34] ■

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Notes

1. References to 'paradigms' and 'scientific revolutions' are based on Thomas Kuhn's book: *The Structure of Scientific Revolutions.*

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14. Glasstone and Dolan, *Effects of Nuclear Weapons*, 71.

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16. Department Of Energy, *United States Nuclear Tests*. Author counted the above ground tests.

17. Staff Officers Field Manual, *Nuclear Weapon Employment*, FM 101-31-1, (February 1963), 88.

18. Staff Officers Field Manual, *Nuclear Weapon Employment*, 88.

19. Ernest Bauer, P*hysics of High Temperature Air, Part II, Applications*, Institute for Defense Analysis, August 1990 6-12.

20. Hans Bethe, “Theory of the Fireball,” Los Alamos Scientific Laboratory, June 17, 1964, 64. Yield is in megatons. Pressure in Bars. R in kilometers. T′ is given as a value at max fireball size (see page 66).

21. Email correspondence with a Defense Threat Reduction Agency, Technical Reachback Physical Scientist and lead Hazard Prediction and Assessment Capability engineer at Applied Research Associates, April 11, 2025.

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32. Spriggs et al., “Fallout Cloud Regimes,” 110.

33. Staff Officers Field Manual, *Nuclear Weapon Employment*, 88.

34. The text references a 1950-version edited by Samuel Glasstone with a slightly different title, *The Effects of Atomic Weapons*. Glasstone mentioned this in the first line of the Preface of teh 1977 version. The title of the first version was adjusted to *Nuclear Weapons* due to Glasstone and Dolan’s point made in 1.11 of the 1977 version; Department Of Energy, *United States Nuclear Tests*, 2-3.